LeRec: A NN/HMM Hybrid for On-Line Handwriting Recognition

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Abstract

We introduce a new approach for on-line recognition of handwritten words written in unconstrained mixed style. The preprocessor performs a word-level normalization by fitting a model of the word structure using the EM algorithm. Words are then coded into low resolution "annotated images" where each pixel contains information about trajectory direction and curvature. The recognizer is a convolution network which can be spatially replicated. From the network output, a hidden Markov model produces word scores. The entire system is globally trained to minimize word-level errors.

1 Introduction

Natural handwriting is often a mixture of different "styles", lower case printed, upper case, and cursive. A reliable recognizer for such handwriting would greatly improve interaction with pen-based devices, but its implementation presents new technical challenges. Characters taken in isolation can be very ambiguous, but considerable information is available from the context of the whole word. We propose a word recognition system for pen-based devices based on four main modules: a preprocessor that normalizes a word, or word group, by fitting a geometrical model to the word structure using the EM algorithm; a module that produces an "annotated image" from the normalized pen trajectory; a replicated convolutional neural network that spots and recognizes characters; and a Hidden Markov Model (HMM) that interprets the networks output by taking word-level constraints into account. The network and the HMM are *jointly* trained to minimize an error measure defined at the word level.

Many on-line handwriting recognizers exploit the sequential nature of pen trajectories by representing the input in the time domain. While these representations are compact and computationally advantageous, they tend to be sensitive to stroke order, writing speed, and other irrelevant parameters. In addition, global geometric features, such as whether a stroke crosses another stroke drawn at a different time, are not readily available in temporal representations. To avoid this problem we designed a representation, called AMAP, that preserves the pictorial nature of the handwriting.

In addition to recognizing characters, the system must also correctly *segment* the characters within the words. To choose the optimal segmentation and take advantage of contextual and linguistic structure, the neural network is combined with a graph-based post-processor, such as an HMM. One approach, that we call INSEG, is to recognize a large number of heuristically segmented candidate characters and combine them optimally with a post-processor (Burges *et al.* 92, Schenkel *et al.* 93). Another approach, that we call OUTSEG, is to delay all segmentation decisions until after the recognition, as is often done in speech recognition. An OUTSEG recognizer must accept entire words as input and produce a sequence of scores for each character at each location on the input. Since the word normalization cannot be done perfectly, the recognizer must be robust with respect to relatively large distortions, size variations, and translations. An elastic word model –e.g., an HMM– can extract word candidates from the network output. The HMM models the long-range sequential structure while the neural network spots and classifies characters, using local spatial structure.

2 Word Normalization

Input normalization reduces intra-character variability, simplifying character recognition. We propose a new word normalization scheme, based on fitting a geometrical model of the word structure. Our model has four "flexible" lines representing respectively the ascenders line, the core line, the base line and the descenders line (see Figure 1). Points $(\mathbf{x}, \mathbf{y})$ on the lines are parameterized as follows:

$$\mathbf{y} = f_j(\mathbf{x}) = k(\mathbf{x} - x_0)^2 + s(\mathbf{x} - x_0) + y_{0j} \tag{1}$$

where k controls curvature, s is the skew, and (x_0,y_0) is a translation vector. The parameters k, s, and x_0 are shared among all four curves, whereas each curve has its own vertical translation parameter y_{0j}. The free parameters of the fit are actually k, s, a (ascenders y_0 minus baseline y_0), b (baseline y_0), c (core line y_0 minus baseline y_0), and d (baseline y_0 minus descenders y_0), as shown in Figure 1. x_0 is determined by taking the average abscissa of vertical extrema points. The lines of the model are fitted to the extrema of vertical displacement: the upper two lines to the vertical maxima of the pen trajectory, and the lower two to the minima. The line parameters $\theta = a, b, c, d, k, s$ are tuned to maximize the joint probability of observed points and parameter values:

$$\theta^* = \underset{\theta}{\arg\max} \ \log P(X \mid \theta) + \log P(\theta) \tag{2}$$

$P(X|\theta)$ is modeled by a mixture of Gaussians (one Gaussian per curve), whose means are the functions of x given in equation 1:

$$P(x_i, y_i \mid \theta) = \sum_{j=0}^{3} w_k N(y_i; f_j(x_i), \sigma_y) \tag{3}$$

where $N(y; \mu, \sigma)$ is the likelihood of y under a univariate Normal model (mean μ, standard deviation σ). The w_k are the mixture parameters, some of which are set to 0 in order to constrain the upper (lower) points to be fitted to the upper (lower) curves. They are computed *a priori* using measured frequencies of associations of extrema to curves on a large set of words. Priors $P(\theta)$ on the parameters (modeled here with Normal distributions) are important to prevent the collapse of the curves. They can be used to incorporate *a priori* information about the word geometry, such as the expected position of the baseline, or of the height of the word. These priors are also used as initial values in the EM optimization of the fit function. The prior distribution for each parameter (independently) is a Normal, with the standard deviation controlling the strength of the prior. In our experiments, these priors were set using some heuristics applied to the data itself. The priors for the curvature (k) and angle (s) are set to 0, while the ink points themselves are preprocessed to attempt remove the overall angle of the word (looking for a near horizontal projection with minimum entropy). To compute the prior for the baseline, the mean and standard deviation of y-position is computed (after rough angle removal). The baseline (b) prior is taken to be one standard deviation below the mean. The core line (c) prior is taken to be two standard deviations above the baseline. The ascender (descender) line prior is taken to be between 1.8 (-0.9) and 3.0 (-2.0) times the core height prior, depending on the maximum (minimum) vertical position in the word.

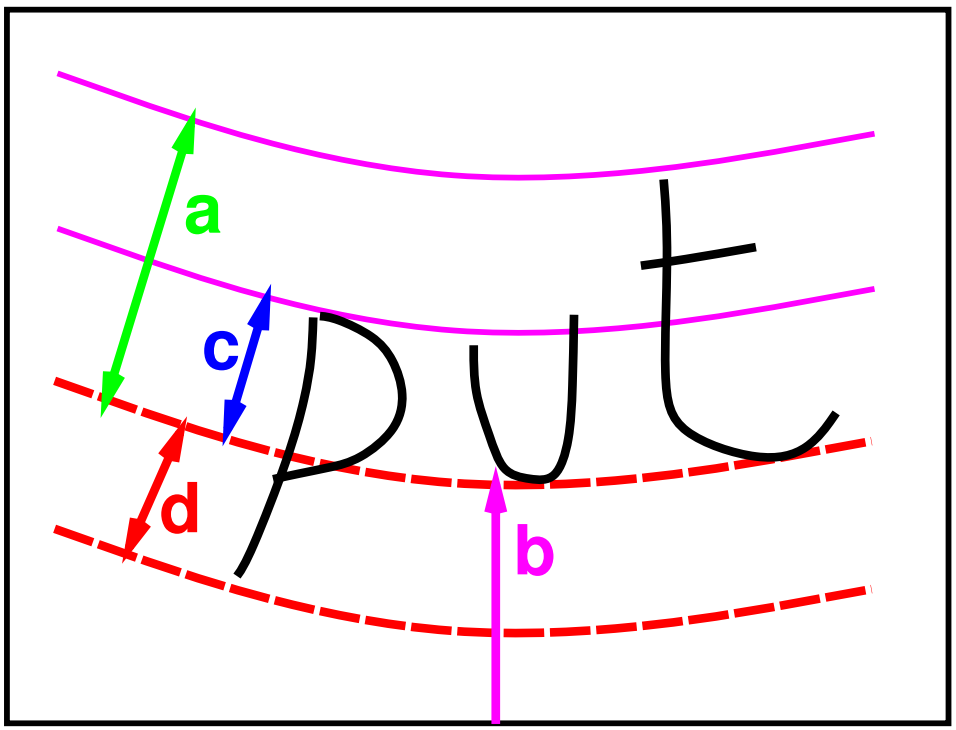

Figure 1: Word Normalization Model: Ascenders and core curves fit *y-maxima* whereas descenders and baseline curves fit *y-minima*. There are 6 parameters: **a** (ascenders curve height relative to baseline), **b** (baseline absolute vertical position), **c** (core line position), **d** (descenders curve position), **k** (curvature), **s** (angle).

The discrete variables that associate each point with one of the curves are taken as hidden variables of the EM algorithm. One can thus derive an auxiliary function which can be analytically (and cheaply) solved for the 6 free parameters θ. Convergence of the EM algorithm was typically obtained within 2 to 4 iterations (of maximization of the auxiliary function).

3 AMAP

The recognition of handwritten characters from a pen trajectory on a digitizing surface is often done in the time domain (Tappert 90, Guyon et al 91). Typically, trajectories are normalized, and local geometrical or dynamical features are sometimes extracted. The recognition is performed using curve matching (Tappert 90), or other classification techniques such as Time-Delay Neural Networks (Guyon et al 91). While these representations have several advantages, their dependence on stroke ordering and individual writing styles makes them difficult to use in high accuracy, writer independent systems that integrate the segmentation with the recognition.

Since the intent of the writer is to produce a legible *image*, it seems natural to preserve as much of the pictorial nature of the signal as possible, while at the same time exploit

the sequential information in the trajectory. We propose a representation scheme, called AMAP, where pen trajectories are represented by low-resolution images in which each picture element contains information about the local properties of the trajectory. An AMAP is a multidimensional array (in our case 5x20x18) obtained by discretizing continuous "feature density" functions (varying smoothly with position (X, Y) and other variables such as direction of motion ϕ) into "boxes". Each of these array elements is assigned a value equal to the integral of the feature density function over the corresponding box. In practice, an AMAP is computed as follows. At each sample on the trajectory, one computes the position of the pen (X, Y) and orientation of the motion ϕ (and possibly other features, such as the local curvature c). Each element in the AMAP is then incremented by the amount of the integral over the corresponding box of a predetermined *point-spread function* centered on the coordinates of the feature vector. The use of a smooth point-spread function (say a Gaussian) ensures that smooth deformations of the trajectory will correspond to smooth transformations of the AMAP. An AMAP can be viewed as an "annotated image" in which each pixel is a feature vector.

A particularly useful feature of the AMAP representation is that it makes very few assumptions about the nature of the input trajectory. It does not depend on stroke ordering or writing speed, and it can be used with all types of handwriting (capital, lower case, cursive, punctuation, symbols). Unlike many other representations (such as global features), AMAPs can be computed for complete words without requiring segmentation. In the experiments we used AMAPs with 5 features at each pixel location: 4 features are associated to four orientations (0^o, 45^o, 90^o, and 135^o); the fifth one is associated to local curvature. For example, when there is a nearly vertical segment in an area, nearby pixels will have a strong value for the first ("vertical") feature. Near endpoints or points of high spatial curvature on the trajectory, the fifth ("curvature") feature will be high. Curvature information is obtained by computing the cosine angle between successive elementary segments of the trajectory. Because of the integration of the point-spread function (a Gaussian), the curvature feature at a given pixel depends on the curvature at different points of the trajectory in the vicinity of that pixel.

4 Convolutional Neural Networks

Image-like representations such as AMAPs are particularly well suited for use in combination with Multi-Layer Convolutional Neural Networks (MLCNN) (Le Cun 89, Le Cun et al 90). MLCNNs are feed-forward neural networks whose architectures are tailored for minimizing the sensitivity to translations, rotations, or distortions of the input image. They are trained to recognize and spot characters with a variation of the Back-Propagation algorithm (Rumelhart et al 86, Le Cun 86).

Each unit in an MLCNN is connected only to a local neighborhood in the previous layer. Each unit can be seen as a local feature detector whose function is determined by the learning procedure. Insensitivity to local transformations is built into the network architecture by constraining sets of units located at different places to use identical weight vectors, thereby forcing them to detect the same feature on different parts of the input. The outputs of the units at identical locations in different feature maps can be collectively thought of as a local feature vector. Features of increasing complexity and scale are extracted by the neurons in

the successive layers.

Because of weight-sharing, the number of free parameters in the system is greatly reduced. Furthermore, MLCNNs can be scanned (replicated) over large input fields containing multiple *unsegmented* characters (whole words) very economically by simply performing the convolutions on larger inputs. Instead of producing a single output vector, such an application of an MLCNN produces a series of output vectors. The outputs detect and recognize characters at different (and overlapping) locations on the input. These multiple-input, multiple-output MLCNNs are called Space Displacement Neural Networks (SDNN) (Matan et al 92, Keeler et al 91).

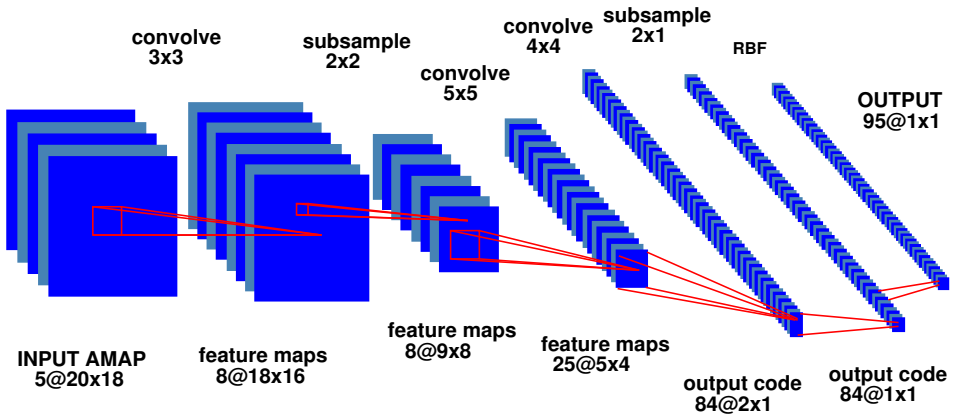

Figure 2: Convolutional Neural Network character recognizer. This architecture is robust to local translations and distortions, with subsampling, shared weights and local receptive fields.

One of the best networks we found for character recognition has 5 layers arranged as illustrated in figure 4:convolution with 8 kernels of size 3x3, layer 2: 2x2 subsampling, layer 3: convolution with 25 kernels of size 5x5, layer 4 convolution with 84 kernels of size 4x4, layer 5: 2x1 subsampling, classification layer: 95 radial basis function (RBF) units (one per class). The subsampling layers are essential to the network's robustness to distortions. The "output code" layer is one (single MLCNN) or a series of (SDNN) 95-dimensional vectors, with a distributed target code for each character corresponding to the weights of the RBF units. The choice of input field dimension was based on the following considerations. We estimated that at least 4 or 5 pixels were necessary for the *core* of characters (between baseline and core line). Furthermore, very wide characters (such as "w") can have a 3 to 1 aspect ratio. On the vertical dimension, it is necessary to leave room for ascenders and descenders (at least one core height each). In addition, convolutional networks perform better with extra borders allowing inputs near the borders to be centered on the input field of some features at the first layer. Finally, the input size must satisfy certain constraints in order to perform the convolutions at each layer (the input size must be equal to a certain value, modulo a step value corresponding to the overall subsampling for the whole network). The last subsampling layer performs a vertical subsampling to make the network more robust to errors of the word normalizer (which tends to create variations in vertical position). Several architectures were tried (but clearly not exhaustively), varying the type of layers (convolution, subsampling) and the number of hidden units. Larger architectures did not necessarily perform better and required considerably more time to be trained. A very small architecture with half the input field also performed worse,

because of insufficient input resolution. Note that the input resolution is nonetheless much less than for optical character resolution, but this is compensated by the additional information available for each pixel on angle and curvature.

In the experiments, the network was first trained with fixed targets (i.e., the RBF centers were fixed). This bootstrap phase was performed on isolated characters. In a second phase, the network was applied to whole words (as described below), and the network weights and RBF centers were tuned in order to optimize a criterion defined with a post-processor at the word level.

5 Segmentation and Post-Processing

The convolutional neural network can be used to give scores associated to characters when the network (or a piece of it corresponding to a single character output) has an input field, called a *segment*, that covers a connected subset of the whole word. A *segmentation* is a sequence of such segments that covers the whole word. Because there are often many possible segmentations, sophisticated tools such as hidden Markov models and dynamic programming are used to search for the best segmentation.

In this paper, we consider two approaches to the segmentation problem called INSEG (for input segmentation) and OUTSEG (for output segmentation). In both approaches the post-processors can be decomposed into two levels: 1) character level scores and constraints obtained from the observations, 2) word level constraints (e.g., from a grammar or dictionary). The INSEG and OUTSEG systems share the second level. The INSEG and OUTSEG architectures are depicted in Figure 3.

In an INSEG system, the network is applied to a number of heuristically segmented candidate characters. A *cutter* generates candidate *cuts*, which represent a potential boundary between two character segments. It also generates *definite cuts*, which we assume that no segment can cross. A *combiner* then generates the candidate segments, based on the cuts found.

The cutter module finds candidate cuts in cursive words (note that the data can be cursive, printed, or mixed). A superset of such cuts is first found, based on the pen velocity along each stroke. Next, several filters are applied to remove incorrect cuts. The filters use vertical projections, proximity to the baseline, and other similar characteristics. Horizontal strokes of "T"s that run into the next character (with no pen up) are also cut here.

Next, the combiner module generates segments based on these cuts. Heuristic filters are again used to significantly reduce the number of candidate segments down to a reasonable number. For example, segments falling across definite cuts, or that are too wide, or that contain too many strokes, are removed from the list of candidates; and segments that contain too little ink are forcibly combined with other segments. Finally, some segments (such as the horizontal or vertical strokes of T's, other vertical strokes that lie geometrically inside other strokes, etc) are also forcibly combined into larger segments.

The network is then applied to each of the resulting segments separately. These scores are then attached to nodes of an *observation graph* in which the connectivity and transition probabilities on arcs represent segmentation and geometrical constraints (e.g., segments must not overlap and must cover the whole word, some transitions between characters are more or less likely given the geometrical relations between their images). Each node in the observation graph thus represents a segment of the input image and a candidate classification for this segment, with a corresponding score or cost.

In an OUTSEG system, all segmentation decisions are delayed until after the recognition, as is often done in speech recognition (Keeler et al 91, Bengio et al 92, Matan et al 92, Schenkel et al 92). The AMAP of the entire *word* is shown to an SDNN, which produces a sequence of output vectors equivalent to scanning the single-character network over all possible pixel locations on the input. The Euclidean distances between each output vector and the targets are interpreted as log-likelihoods of the output given a class. To construct an *observation graph*, we use a set of character HMMs, modeling the sequence of network outputs observed for each character. We used three-state HMMs for each character, with a left and right state to model transitions and a center state for the character itself. The observation graph is obtained by connecting these character models, allowing any character to follow any character.

On top of the constraints given in the observation graph, additional constraints that are independent of the observations are given by what we call a *grammar graph*, which can embody lexical constraints. These constraints can be given in the form of a dictionary or of a character-level grammar (with transition probabilities), such as a trigram. Recognition searches the best path in the observation graph that is compatible with the grammar graph. When the grammar graph has a complex structure (e.g. a dictionary), the product of the grammar graph with the observation graph can be huge. To avoid generating such a large data structure, we define the nodes of this product graph procedurally and we only instantiate nodes along the paths explored by the graph search (and pruning) algorithm.

With the OUTSEG architecture, there are several ways to put together the *within-character constraints* of the HMM observation graph with the *between-character constraints* of the grammar graph. The approach generally followed in HMM speech recognition systems consists in taking the product of these two graphs and searching for the best path in the combined graph. This is equivalent to using the costs and connectivity of the grammar graph to connect together the character HMM models from the observation graph, i.e., to provide the transition probabilities *between* the character HMMs (after making duplicates of the character models for each corresponding character in the grammar graph). Variations of this scheme include pruning the search (e.g. with beam search) and separating the search in the observation graph and the grammar graph. The latter idea can be implemented by first extracting likely segmentations (or a lattice of character hypotheses) from the observation graph (character HMMs), and using those to construct and INSEG-like observation graph (where each node corresponds to a character hypothesis for a segment). The approach used in the INSEG architecture can then be used.

A crucial contribution of our system is the joint training of the neural network and the post-processor with respect to a single criterion that approximates word-level errors. We used the following *discriminant* criterion: minimize the total cost (sum of negative log-likelihoods) along the "correct" paths (the ones that yield the correct interpretations), while maximizing the costs

of *all* the paths (correct or not). The discriminant nature of this criterion can be shown with the following example. If the cost of a path associated to the correct interpretation is much smaller than all other paths, the criterion is very close to 0 and almost no gradient is back-propagated. On the other hand, if the lowest cost path yields an incorrect interpretation but differs from a path of correct interpretation on a sub-path, then very strong gradients will be propagated along that sub-path, whereas the other parts of the sequence will generate almost no gradient. Within a probabilistic framework, this criterion corresponds to maximizing the mutual information (MMI) between the observations and the correct interpretation (Nadas et al 88). The mutual information $I(C,Y)$ between the correct interpretation C (sequence of characters) and transformed observations Y (sequence of outputs of the last layer of the neural net before the RBFs) can be rewritten as follows, using Bayes' rule:

$$I(C,Y) = \log \frac{P(Y,C)}{P(Y)P(C)} = \log \frac{P(Y|C)}{P(Y)} \tag{4}$$

where $P(Y|C)$ is the likelihood of transformed observations Y constrained by the knowledge of the correct interpretation sequence C, $P(Y)$ is the unconstrained likelihood of Y (i.e., taking all interpretations possible in the model into account) and $P(C)$ is the prior probability of the sequence of characters C. Interestingly, when the class priors are fixed, maximizing $I(C,Y)$ is equivalent to maximizing the posterior probability of the correct sequence C, given the observations Y (also known as the MAP criterion):

$$P(C|Y) = \frac{P(Y|C)P(C)}{P(Y)} \tag{5}$$

Both the MMI and MAP criteria are more discriminant than the maximum likelihood criterion (maximizing $P(Y|C)$) because the parameters are used not to *model* the type of observations corresponding to a particular class C, but rather to *discriminate* between classes. The most discriminant criterion is the number of classification errors on the training set but unfortunately it is computationally very difficult to directly optimize such a discrete criterion.

During global training, the MMI criterion was optimized using enhanced (LeCun 89) stochastic gradient descent with respect to *all* the parameters in the system, most notably the network weights. Experiments described in the next section have shown important reductions in error rates when training with this word-level criterion instead of just training the network separately for each character. Similar combinations of neural networks with HMMs or dynamic programming have been proposed in the past, for speech recognition problems (Bengio et al 92).

6 Experimental Results

In the first set of experiments, we evaluated the generalization ability of the neural network classifier coupled with the word normalization preprocessing and AMAP input representation. All results are in *writer independent* mode (different writers in training and testing). Tests on a database of isolated characters were performed separately on four types of characters: upper case (2.99% error on 9122 patterns), lower case (4.15% error on 8201 patterns), digits

(1.4% error on 2938 patterns), and punctuation (4.3% error on 881 patterns). Experiments were performed with the network architecture described above. Additional training data was generated by applying local affine transformations to the original pen trajectories, thus improving the robustness of the recognizer (see (Drucker et al 93) for a similar algorithm applied to optical character recognition).

The second and third set of experiments concerned the recognition of lower case words (writer independent). The tests were performed on a database of 881 words. First we evaluated the improvements brought by the word normalization to the INSEG system. For the OUTSEG system we *have* to use a word normalization since the network sees a whole word at a time. With the INSEG system, and before doing any word-level training, we obtained without word normalization 7.3% and 3.5% word and character errors (adding insertions, deletions and substitutions) when the search was constrained within a 25461-word dictionary. When using the word normalization preprocessing instead of a character level normalization, error rates dropped to 4.6% and 2.0% for word and character errors respectively, i.e., a relative drop of 37% and 43% in word and character error respectively.

In the third set of experiments, we measured the improvements obtained with the joint training of the neural network and the post-processor with the word-level criterion, in comparison to training based only on the errors performed at the character level. Training was performed with a database of 3500 lower case words. For the OUTSEG system, without any dictionary constraints, the error rates dropped from 38% and 12.4% word and character error to 26% and 8.2% respectively after word-level training, i.e., a relative drop of 32% and 34%. For the INSEG system and a slightly improved architecture, without any dictionary constraints, the error rates dropped from 22.5% and 8.5% word and character error to 17% and 6.3% respectively, i.e., a relative drop of 24.4% and 25.6%. With a 25461-word dictionary, errors dropped from 4.6% and 2.0% word and character errors to 3.2% and 1.4% respectively after word-level training, i.e., a relative drop of 30.4% and 30.0%. Even lower error rates can be obtained by drastically reducing the size of the dictionary to 350 words, yielding 1.6% and 0.94% word and character errors.

The AMAP preprocessing with two-dimensional multilayer convolutional network was also compared with another approach developed in our laboratory by Guyon et al. (1991), based on a time-domain representation and one-dimensional convolutional network (or Time-Delay Neural Network). The networks were not trained on the same data, but were both tested on the same database of 17858 isolated characters developed by AT&T GIS for comparing a variety of commercial character recognizers with the recognizers developed in our laboratory. Error rates for the AMAP network were 2.0%, 5.4%, 6.7% and 2.5% on digits, upper case, lower case and a reduced set of punctuation symbols. On the same categories, the Time-Delay Neural Network (based on a temporal representation) attained rates of 2.6%, 6.4%, 7.7% and 5.1% errors, respectively. However, we noticed that the two networks often *made errors on different patterns*, probably because they are based on different input representations. Hence we combined their output (by a simple sum), and obtained on the same classes 1.4%, 3.9%, 5.3% and 2.2% errors, i.e., a very important improvement. This can be explained because these recognizers not only make errors on different patterns, but also have good rejection properties: the highest scoring class tends to have a low score when it is not the correct class.

NCR (AT&T GIS) conducted a test in which such a combined system was compared with

4 commercial classifiers on the printable ASCII set (isolated characters, including upper and lower case, digits, and punctuations). On this benchmark task, because characters are given in isolation, there are inherent confusions between many sets of characters such as ("O","o"), ("P","p"), ("2","z","Z"), ("l","i" with no dot, "1","|"), etc... We estimated that the best one could hope because of these confusions was around 12.9% error rate (by not counting these confusions as errors with our best recognizer). Our recognizer attained an 18.9% error rate, that is 6% worse than this estimated floor. The error rates obtained by the commercial recognizers were, in decreasing order of performance, 30.8%, 32.5%, 34.0%, and 39.0%. These are respectively 17.9%, 19.6%, 21.1%, and 26.1% above our estimated floor. These results are illustrated in the bar chart of Figure 4.

7 Conclusion

We have demonstrated a new approach to on-line handwritten word recognition that uses word or sentence-level preprocessing and normalization, image-like representations, convolutional neural networks, graph-based word models, and global training using a highly discriminant word-level criterion. Excellent accuracy on various writer independent tasks were obtained with this combination.

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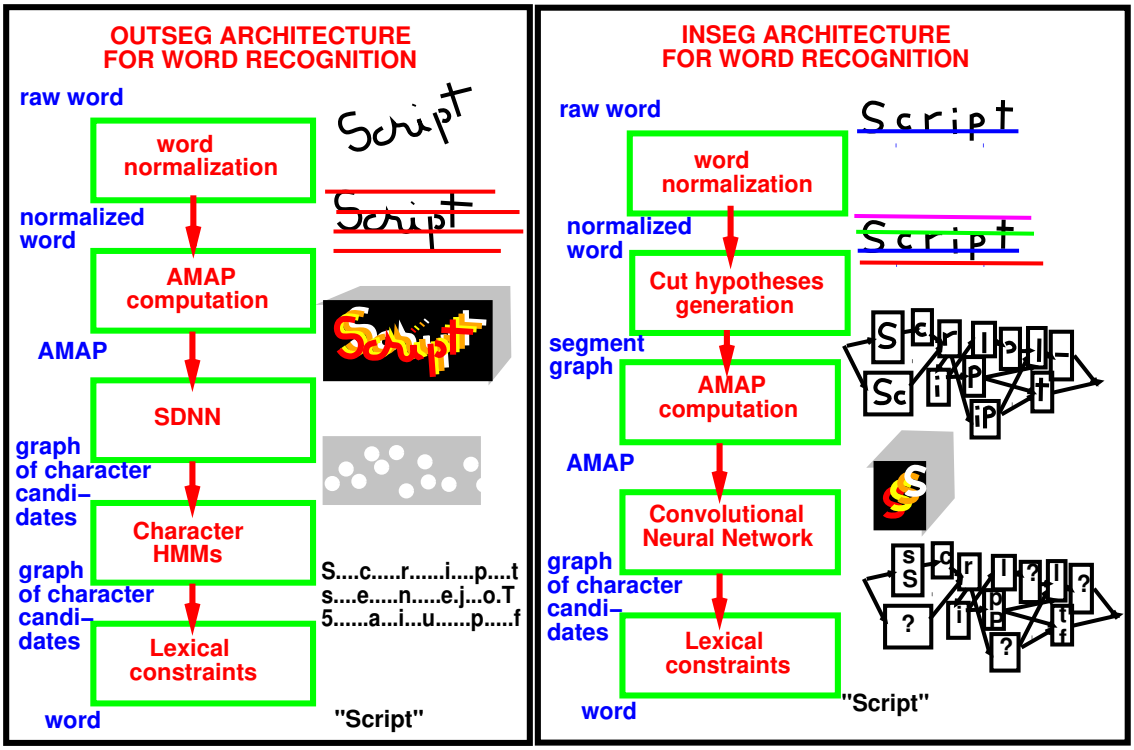

Figure 3: INSEG and OUTSEG architectures for word recognition.

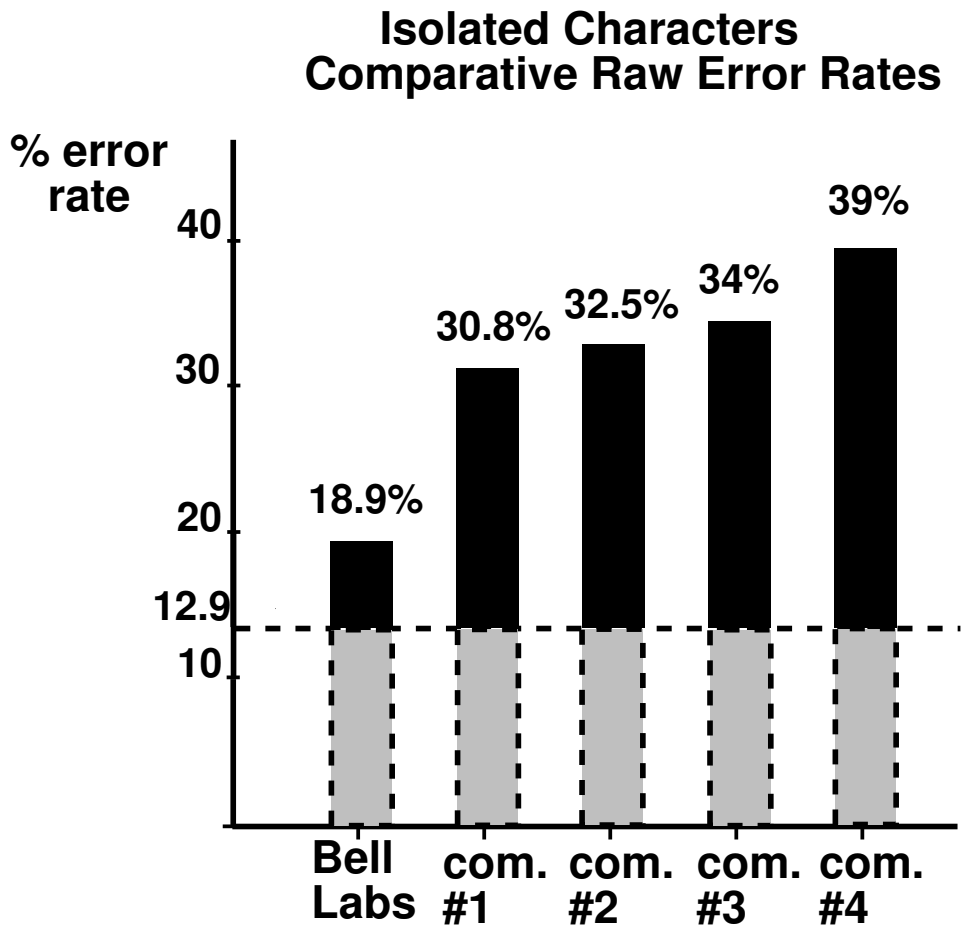

Figure 4: Comparative results on a benchmark test conducted by NCR (AT&T GIS) on isolated character recognition (uppers, lowers, digits, symbols). The last five bars represent the results obtained by five competing commercial recognizers. The floor (12.9%) represents the best result we could obtain by not counting natural confusions as errors.